1. **Average Velocity.** An object moves in one dimension with a velocity in m/s given by $v(t) = 8\sin(\pi t) + 2t$. Find its average velocity over the time interval from $t = 0$ to $t = 10$ seconds.

2. $\sin^2$ **and** $\cos^2$ **Integrals.** Use substitution to verify that

$$\int \sin^2 ax\, dx = \frac{x}{2} - \frac{\sin(2ax)}{4a} + C$$
$$\int \cos^2 ax\, dx = \frac{x}{2} + \frac{\sin(2ax)}{4a} + C$$

3. **Shifting Functions.** Show that the net change is not changed by shifting the function along the direction of integration by verifying

$$\int_a^b f(x+c)\,dx = \int_{a+c}^{b+c} f(u)\,du.$$

4. **Scaling Functions.** Show that the net change scales with the variable of integration by verifying

$$\int_a^b f(cx)\,dx = \frac{1}{c}\int_{ac}^{bc} f(u)\,du.$$

5. **Repeated Substitution.** Evaluate the following integrals by repeated applying substitution

   (a) $\displaystyle\int \frac{dx}{\sqrt{1+\sqrt{1+x}}}$

   (b) $\displaystyle\int_0^{\pi/2} \frac{\cos x \sin x}{\sqrt{\cos^2 x + 16}}\, dx$

6. **Equal area properties of parabolas.** Consider the parabola $y = x^2$ with points P, Q, and R anywhere between P and Q. Let l_P, l_Q, and l_R be the lines tangent to the parabola at P, Q, R respectively. Let P' be the intersection of l_Q and l_R, Q' the intersection point of l_P and l_R and R' the intersection point of l_P and l_Q. Show that the area of the triangle ΔPQR is twice the area of $\Delta P'Q'R'$.

